Title: The King's Rule

Brief Overview:

This lesson teaches finding missing data on function tables, generating rules for function tables, and graphing data from function tables on coordinate grids. It is expected that students have prior experience with identifying numeric patterns on a function table, using variables to represent the rule for a function, and graphing ordered pairs on a coordinate grid.

During this lesson each student will create, analyze, and construct functions that are based around the story The King's Chessboard by David Birch.

NCTM Content Standard/National Science Education Standard:

Understand patterns, relations, and functions
- Represent and analyze patterns and functions, using word, tables, and graphs.

Represent and analyze mathematical situations and structures using algebraic symbols
- Represent the idea of a variable as an unknown quantity using a letter or a symbol.
- Express mathematical relationships using equations.

Use mathematical models to represent and understand quantitative relationships
- Model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.

Analyze change in various contexts
- Investigate how a change in one variable relates to a change in a second variable.

Grade/Level:

Grade 5

Duration/Length:

Three class sessions, approximately 60 minutes per session

Student Outcomes:

Students will:

- Create a function table for a given situation
- Complete a function table with a given rule
- Generate a rule for a function table
- Construct a graph using data in a function table

Materials and Resources:

Day 1
- Function Table Questions (Teacher Resource 1)
- Dry erase boards and markers
- The King's Chessboard by David Birch
- Copy of a chessboard (Student Resource 1)
- The King's Rule Is… Transparency (Teacher Resource 2)
- The King's Rule (Student Resource 2 and Teacher Resource 3)
- Function Machine (Student Resource 3)
- Digit Cards (Student Resource 4)
- Input/Output Cards (Teacher Resource 4 A-E)
- Function Machine Box

Day 2
- The King's Chessboard by David Birch
- Dry erase boards and markers
- The Queen's Rule (Teacher Resource 5)
- Student copy of The Queen's Rule (Student Resource 5 and Teacher Resource 6)
- Chess Match (Student Resource 6 A-C)
- Exit Ticket (Student Resource 7 and Teacher Resource 7))

Day 3
- The game Battleship
- The Royal Kingdom (Student Resource 8)
- Royal Palace Plots (Student Resource 9 and Teacher Resource 8)
- The King's Royal Path (Teacher Resource 9)
- Summative Assessment (Student Resource 10 A-C and Teacher Resource 10
- A-C)

Development/Procedures:

Day 1
- o Pre-assessment
  Pose the question: What is a function table? Tell students to brainstorm possible responses with a partner.

  Facilitate a whole group discussion using Teacher Resource 1 as a guide for the discussion and allow students use of response boards to record individual answers.

- o Engagement
  Read the story, The King's Chessboard to the students and stop at the end of page 6 after the Queen suggests that the King ask the wise man how much rice it is in all.

- o Exploration
  Distribute a copy of a chessboard to each student to use as a reference (Student Resource 1).

  Pose the question: Do you think the King will be satisfied with the reward that he has given the wise man? Why or why not? Give the students time to discuss. Hint: you may want to tell students to use math calculations to justify their responses with a partner.

  Consider starting the beginning of the pattern for those students who are having difficulty: 2, 4, 8, 16, 32.

- o Explanation
  Have students share initial conclusions about the King's satisfaction with the deal that he made with the wise man.

  Finish reading the story to the students. Give the students the following scenario: Pretend the wise man went to the King of another land and was offered a reward. This time he asks to receive triple the amount of rice each day for each square of the chessboard. He began on day one with three grains of rice. Create a function table with the students to show how much rice he would have on the tenth day using Teacher Resource 2 as a transparency. Make sure to write the rule at the top of Teacher Resource 2.

  Give the students the following scenario: The Weigher of the King's Grain needed more rice, so he went to the market. He found that he needed a bag to carry the rice. The cost of the bag was $2.00. Each bag of rice could hold $3.50 worth of rice. Create a function table (Teacher Resource 2) with the students to show how much it would cost for two bags, three bags, and eight bags of rice. Again, make sure to write the rule at the top of Teacher Resource 2.

  Guide the students toward independent work by completing an additional example from Teacher Resource 2. Distribute Student Resource 2 and complete the first two problems with the students as guided practice.

- o Application
  Have students complete Student Resource 2 individually or in pairs. Review answers to the function tables (Teacher Resource 3).

- o Differentiation
  - ▪ Reteach
    Pull students in a small group and distribute the Function Machine (Student Resource 3) and digit cards (Student Resource 4). Have students place a digit

card in the input side of the function machine. Give students a rule and have them place the correct digit card in the output side. Repeat as needed.

- Enrich
  Ask the students to create a scenario where a function table could be used to find the outcome of a given event. Have a partner create a function table that represents the scenario.

o Assessment
Show students an input number and give them the rule from Teacher Resource 4 A-E. Give the students the rule and have students write down the output for that number on a response board and hold up their responses. Place the input card in the function machine and show the output. (Directions for making a function machine can be found at http://www.uen.org/Lessonplan/preview.cgi?LPid=18873. Scroll down to materials and click on Function Boxpdf).

Day 2
o Engagement
Refer back to The King's Chessboard, and re-read pages 6 and 7 to the class. State that the Queen realized that the King's pride was getting in the way of his wisdom. The Queen always seemed to be stronger in math and had a good idea of what was happening.

Now share the following trick, stating that it is one of the Queen's wise math tricks known as "The Queen's Royal Number."   Ask one student to pick a number and share it with the class but not with you. Have all students use a white board to do the following:
- Multiply the number by 5.
- Add 6 to the product.
- Multiply the sum by 4.
- Add 9 to the product.
- Multiply the sum by 5.
- Ask what is the final result?

You can determine the original number by doing   the following:
- Subtract 165 from the answer.
- Drop the 0s in the ones and tens place.
- This is the original number that can be shared with the class.
- Tell the students that the Queen knew your original number because she had a secret algebraic rule that she followed.

o Exploration
Show students Teacher Resource 5 and ask them to work with a partner to find the Queen's rule for completing the function table.

o Explanation
Allow students to share their responses. Continue to use Teacher Resource 5 to explain the patterns and rule of the function table (12x).

To practice finding the rule with missing numbers in the function table, work through Student Resource 5 with the students. Review writing the rule for a function table as an eaquation if necessary. Point out any patterns or techniques that the students haven't already found. Answers can be found on Teacher Resource 6.

o Application
Have students play "Chess Match" (Student Resource 6A-B) with a partner.
- Have students place cards face down.
- Tell students that one color of cards has function tables on them and the other color of cards has the rules on them.
- Students should pick one card from each color and look at them and decide if the rule matches the function table.
- If the rule matches, the student gets to keep the match and go again.
- If the rule does not match, they must flip the cards over and it is the next person's turn.
- The students who are not choosing must check to make sure that their partners are correct.
- The person with the most matches at the end wins.

o Differentiation
- Reteach
Visit the website http://www.mathplayground.com/FunctionMachine.html with students who need extra support finding the rule for a given function table.
- Enrich
Have students create a function table using a rule of their choice. Students can then switch function tables with a partner to try and figure out the rule.

o Assessment
Have students complete an exit ticket (Student Resource 7, "Royal Exit") to turn in as they leave. Answers can be found on Teacher Resource 7.

Day 3
o Engagement
Display the game Battleship. Say to students that you are pretending to hide the King on one of the ships. Tell them that they must give you coordinates to try and find on which ship hecan be found. Inform them that the King cannot be released until all coordinates on the ship where the King is located have been hit.

- o Exploration
  Give students "The Royal Kingdom" (Student Resource 8) and tell them that the King was so frustrated with his bad decision that he has decided to run away. However, the town's supply of rice is running low, so we need their help to find him. (Note: You may need to number the coordinate grid before reproducing the resource.)

- o Explanation
  Have students share ideas of how to find the King. Review all of the parts of a coordinate grid (origin, *x*-axis, *y*-axis), and note them on a transparency of Student Resource 8. Also review how to plot an ordered pair.

  Model how to complete number 1 on "Royal Palace Plots" (Student Resource 9) with the students, explaining how to plot the *x* and *y* coordinates from the function table onto the coordinate grid. Guide the students as they complete number two on "Royal Palace Plots (Student Resource 9). Answer key can be found on Teacher Resource 8.

  Review answers with students.

- o Application
  Redirect the student's attention to "The Royal Kingdom" (Student Resource 8). Tell the students that the royal tracker has been searching for the King and noticed that the king was walking in a pattern. For every 2 steps he went to the right, he went 3 steps up. Show Teacher Resource 9 to display the function table that shows the King's path. Track the King to his royal hide-out which is located at the point where he has gone a total of 12 steps to the right and 18 steps up.

  Review findings with students

- o Differentiation
  - ▪ Reteach
    Monitor student progress and pull those students who are having difficulty to help complete "The Royal Kingdom" (Student Resource 8)
  - ▪ Enrich
    Have students work with a partner to generate a function table and then use "The Royal Kingdom" (Student Resource 8) to plot those coordinates which can be called trees or royal landmarks.

Summative Assessment:

The students will be presented with a variety of function tables. They will be required to fill in missing values and generate a rule for a particular function table. The students will also be given a completed function table in which they will have to be able to graph the

function on a coordinate grid. This assessment included an extended constructed response item in which they will complete the function table for a two-step rule and explain how changing a rule effects the data (Student Resource A-C). Answer key can be found on Teacher Resource 10 A-C.
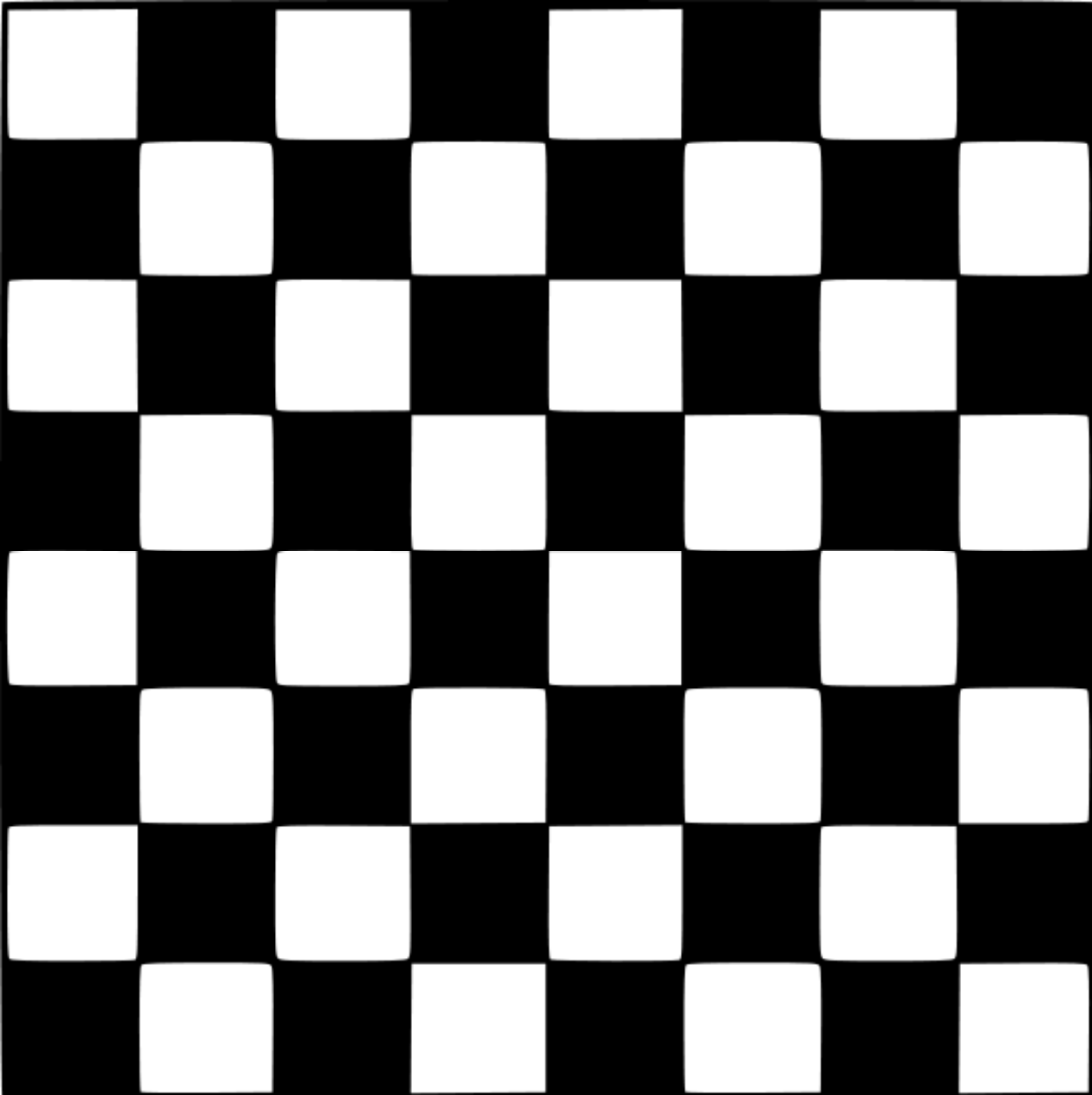
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The King's Rule

A game of chess costs $7.00. Fill in the function table to show how much 12 games of chess would cost.

Rule: $n \bullet 7 = y$

Input x	Output y
1	$7.00
2	
3	
4	
5	$35.00
6	
7	
8	
9	
10	
11	
12	

What mathematical ideas helped you to complete the function table?

_____

_____

_____

_____

Rule: $x - 0.5 = y$

Input X	Output y
30	29.5
25	
20	
	14.5
	9.5
0.5	

Rule: $x + 17 = y$

Input x	Output y
3	20
7	
	27
11	
	30
14.7	

Rule: $(x \div 2) + 3 = y$

Input x	Output y
2	4
6	
10	
	9
	10.5
40	
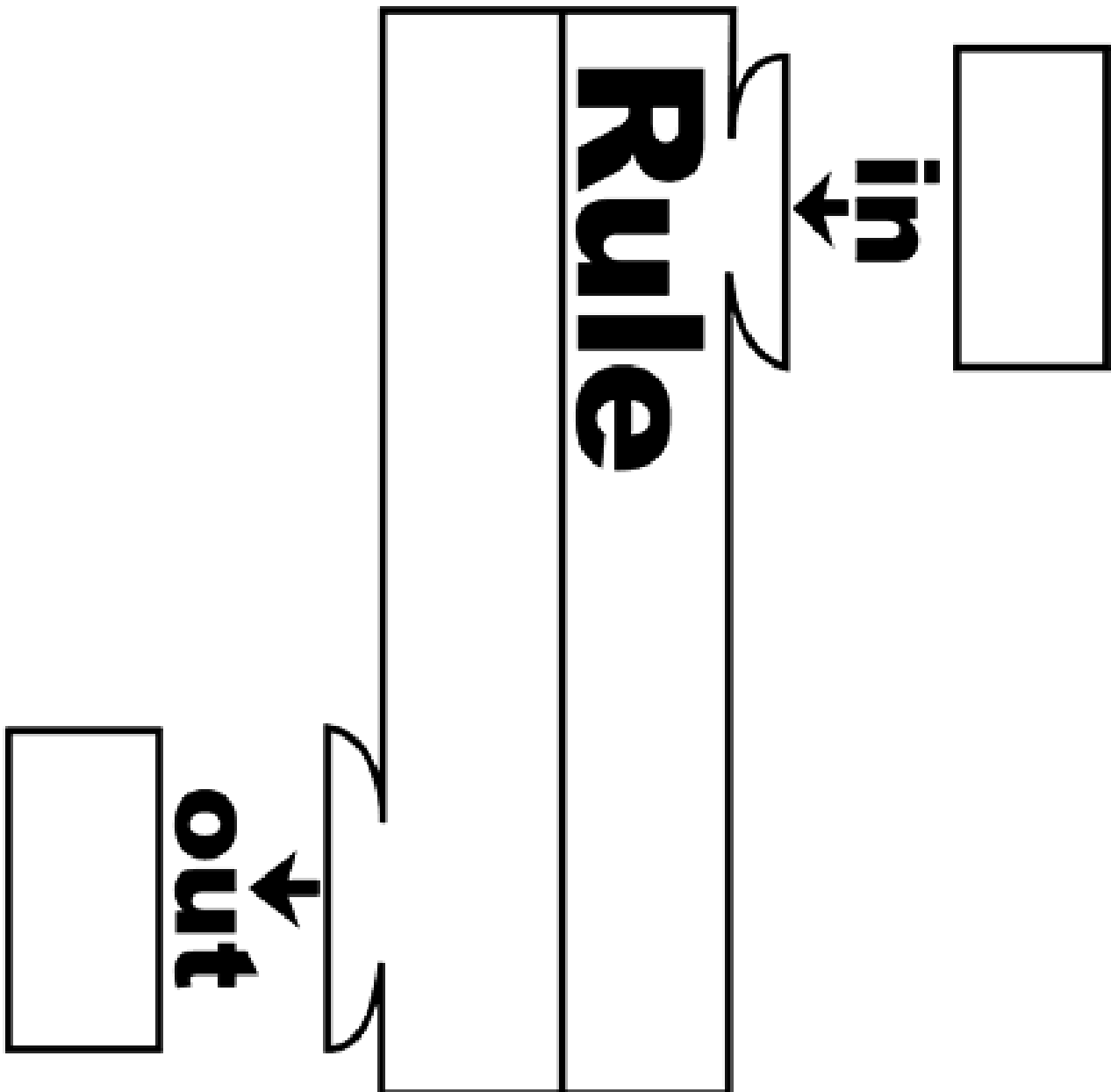
1	2	3
4	5	6
7	8	9
	0	

The Queen's Rule

Input x	Output y
14	16.05
55.1	57.15
	75.25
	102.05
153.8	
	172.04

Rule:_____

Input x	Output y
3.52	3.6
45.02	45.1
	758.41
977.99	
	989.2
1,293.69	

Rule:_____

Input x	Output y
183	61
150	50
144	
	32
	27
45	
45	

Rule:_____

Input x	Output y
3	9
5	17
7	
	33
12	
	77

Rule: _____

When making the cards for the game, place the chess piece pictures on one side and the function tables on the other. Do this again for the rules. Makes sure that the rules and the function tables are different colors.

The Chess Match
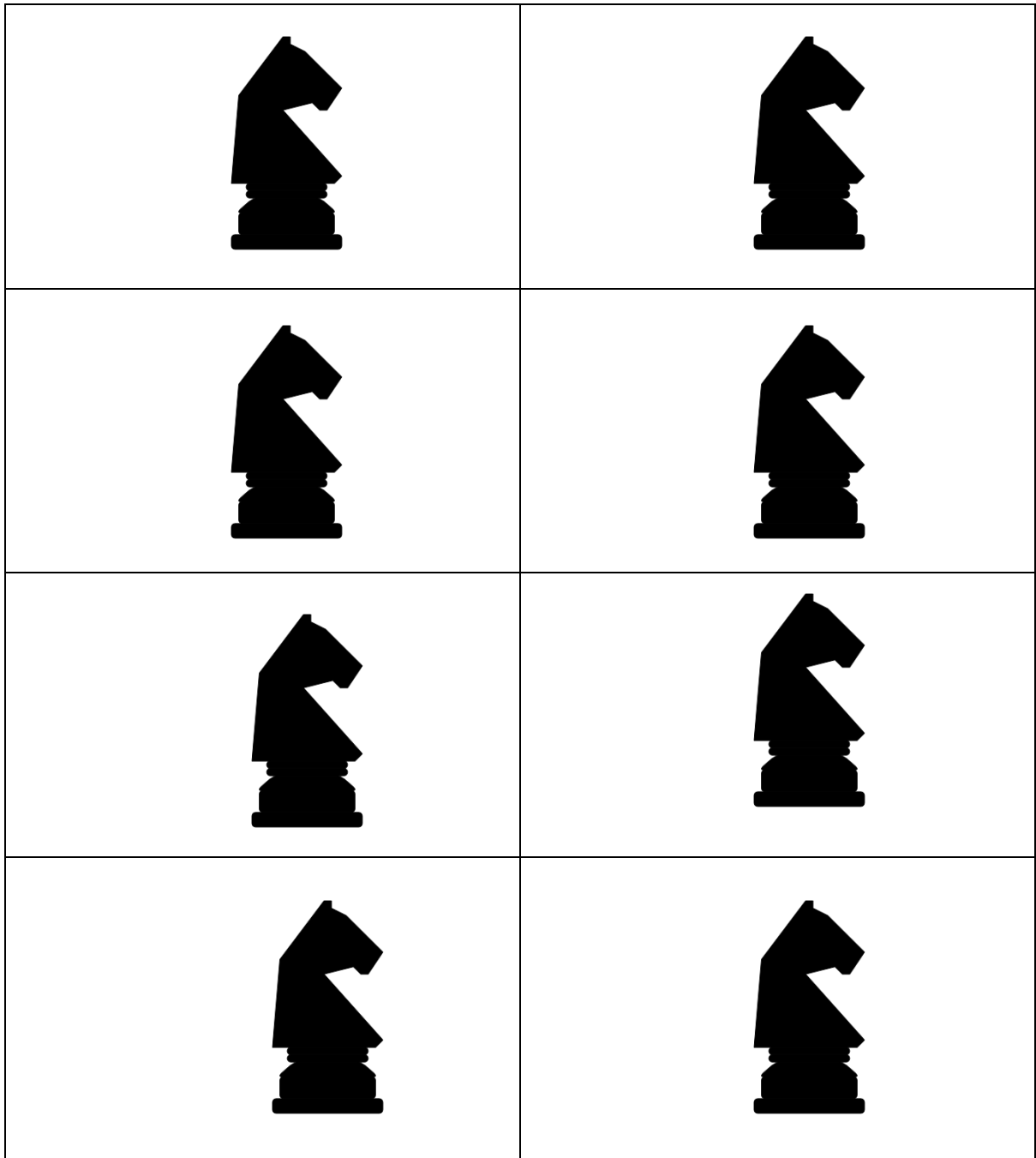
The Chess Match

x	y
1	4
2	5
6	9
7	10
8	11
10	13

x	y
1	0
5	4
8	7
9	8
10	9
11	10

x	y
1	2
2	4
3	6
7	14
10	20
11	22

x	y
3	13
4	14
5	15
9	18
10	20
20	30

x	y
8	2
24	6
36	9
40	10
44	11
56	14

x	y
3	3.3
2.7	3.3
6	6.2
8	8.3
10.4	10.7
11.8	12.1

x	y
13.8	13.2
14.2	13.6
15.9	15.3
20	19.4
21.3	20.7
41	40.4

x	y
3	36
4	48
8	96
10	120
12	144
20	240

The Chess Match

Rule: $n + 3$	**Rule:** $n - 1$
Rule: $n \bullet 2$	**Rule:** $n + 10$
Rule: $n \div 4$	**Rule:** $n + 0.3$
Rule: $n - 0.6$	**Rule:** $n \bullet 12$

Royal Exit

x	9	11	?	17
y	27	33	45	?

What is the rule? Use what you know about functions and patterns to fill in the missing values.

_____

_____

_____

The Royal Kingdom

y

x

Royal Palace Plots

1.

x	y
2	4
3	5
5	7
10	12

y

x
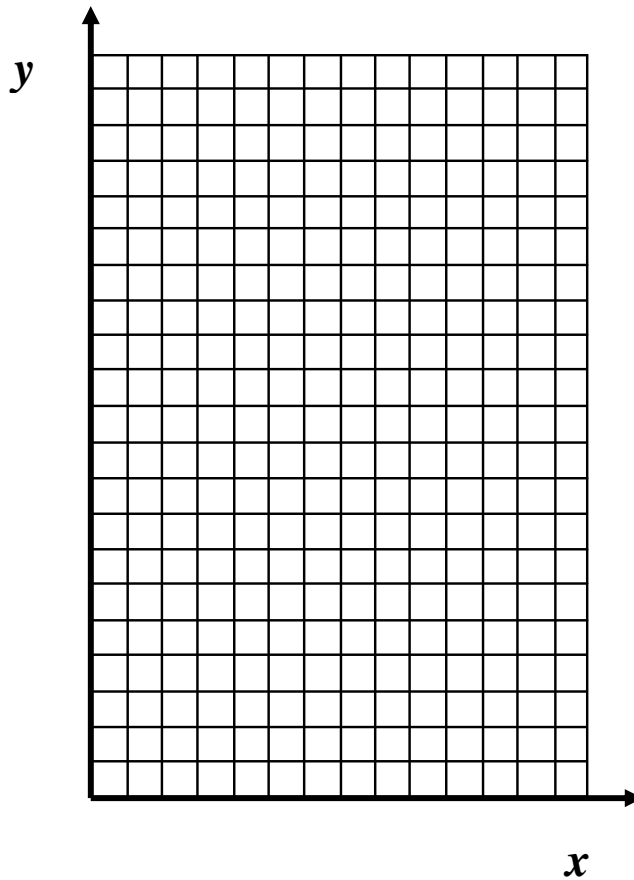
2.

x	y
0	1
1	3
4	9
8	17

Rule: _____

y

Royal Mastery

x
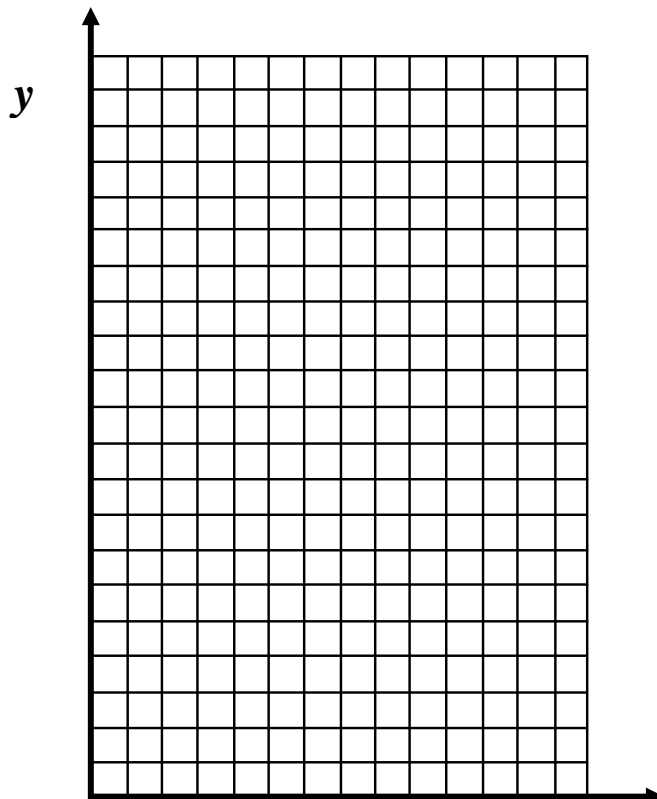
1. Use the rule to complete each function table.

Rule: x ÷ 2

Input x	Output y
6	3
10	
	7
20	
	12
30	

Rule: (x – 4) • 3

Input x	Output y
5	3
6	
7	
10	
15	
34	

2. Identify the rule.

Input x	Output y
450.2	450.9
500.7	501.4
650.5	651.2
750	750.7

Rule:_____

3. Use the function table to graph the data on a coordinate grid.
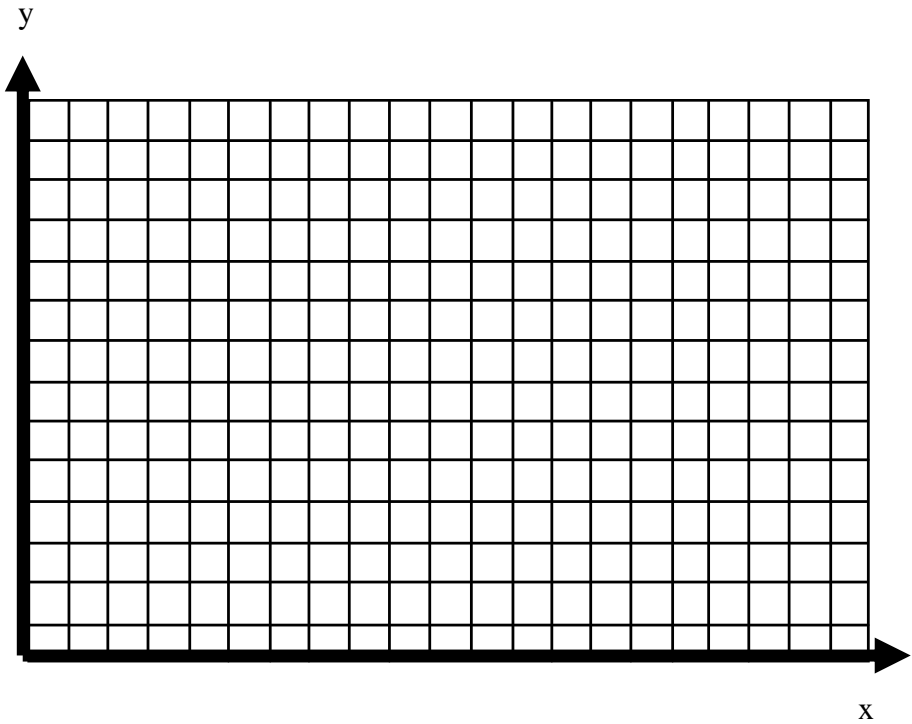
X	y
0	3
1	4
2	5
3	6
4	7
7	10

y

x

Brief Constructed Response

Step A

Complete the function table.

Rule: $2 \bullet x + 5 = y$

Input x	Output y	Ordered Pair (x, y)
2	9	
4		
6		
8		

Step B

- Explain why your answer is correct. Use what you know about patterns and function tables in your explanation. Use words, numbers, and or symbols in your explanation.

_____

_____

_____

_____

_____

_____

Pre-Assessment Questions

What is a function table?

How do you read a function table?

What are some ways to write a rule for a function table?

What does the *x* and *y* represent on a function table?

What is the relationship between input and output on the function table?

The King's Rule Is: _____

Input (X)	Output (Y)

The King's Rule

A game of chess costs $7.00. Fill in the function table to show how much 12 games of chess would cost.

Rule: $7x = y$

Input x	Output Y
1	$7.00
2	$14.00
3	$21.00
4	$28.00
5	$35.00
6	$42.00
7	$49.00
8	$56.00
9	$63.00
10	$70.00
11	$77.00
12	$84.00

What mathematical ideas helped you to complete the function table?

Since each game of chess cost $7.00, I knew that I had to multiply the input (number of games) by $7.00 (cost of one game) to get the total cost.

Rule: $x - 0.5 = y$

Input x	Output y
30	29.5
25	24.5
20	19.5
15	14.5

Rule: $x + 17 = y$

Input x	Output y
3	30
7	24
10	27
11	28
13	30
14.7	31.7

Rule: $(x \div 2) + 3 = y$

Input x	Output y
2	4
6	6
10	8
12	9
15	10.5
40	23

Input and Output Cards For The Function Machine

These numbers should be placed on an index card with the input on the front of the card and the output on the back of the card. The rule is listed for your information only, and should not be placed on the card.

Rule: $n + 4$

2	6

5	9

Rule: n+4

7.5	11.5
14.2	18.2

Rule: $10n - 1$

3	29
5	49

4	1.5
5	2.5
8	5.5

| 11 | 8.5 |

x	*y*
1	12
3	36
4	48
6	72
12	144

The Queen's Rule

Rule: x + 2.5

Input x	Output y
14	16.5
55.1	57.6
73.2	75.7
100	102.5
153.8	156.3
169.9	172.4

Rule: x + 0.8

Input x	Output y
3.5	4.3
45.0	45.8
758.3	759.1
977.9	978.7
989.1	989.9
1,293.6	1294.4

Rule: x ÷ 3

Input x	Output y
183	61
150	50
144	48
96	32
81	27
45	15

Rule: 4x

Input x	Output y
3	12
5	20
7	28
9	36
12	48
20	80

Royal Exit

x	9	11	?	17
y	27	33	45	?

What is the rule? Use what you know about functions and patterns to fill in the missing values.

Rule: 3x   (Missing Values $x = 15, y = 51$) Accept any reasonable response.

Royal Palace Plots

1.

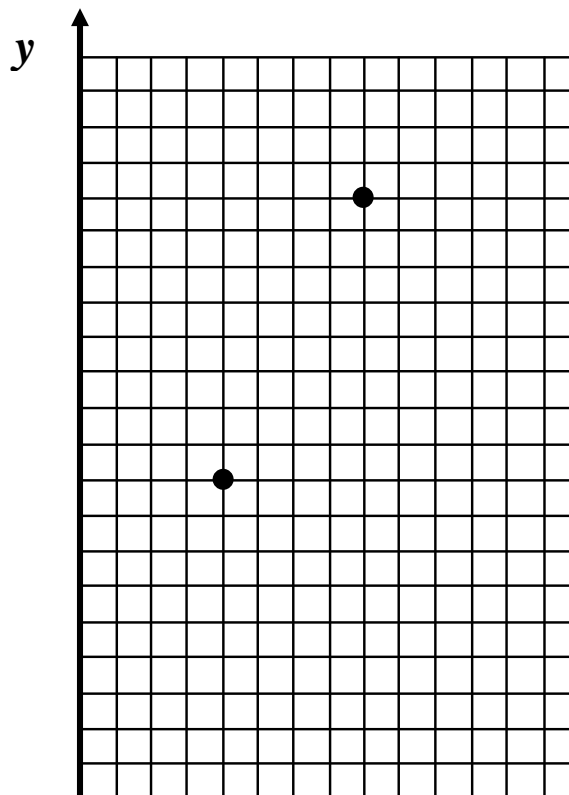| x	y
2	4
3	5
5	7
10	12
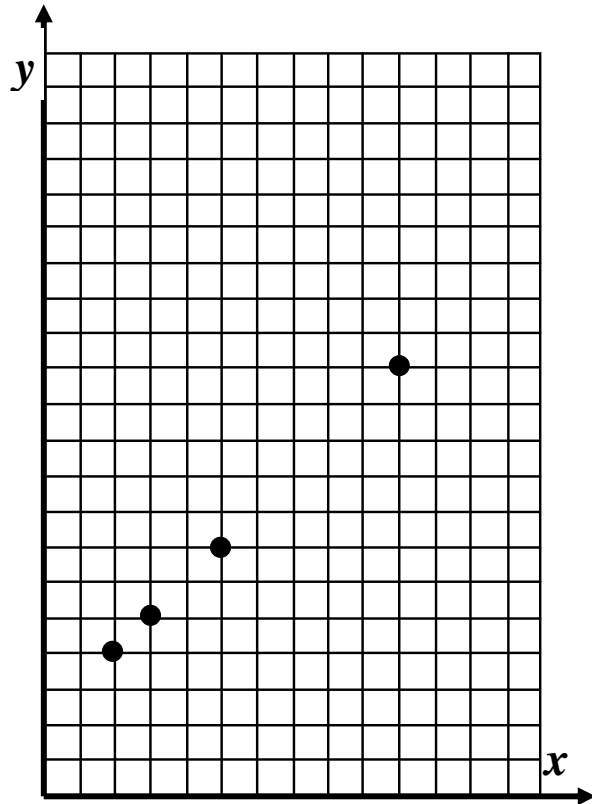
2.

x	y
0	1
1	3
4	9
8	17

The King's Royal Path

x	
2	3
4	6
6	9
8	12
10	15
12	18

Royal Mastery

1. Use the rule to complete each function table.

Rule: x ÷ 2

Input x	Output y
6	3
10	5
14	7
20	10
24	12
30	15

Rule: (x – 4) • 3

Input x	Output y
5	3
6	6
7	9
10	18
15	33
34	90

2. Identify the rule.

Input x	Output y
450.2	450.9
500.7	501.4
650.5	651.2
750	750.7

Rule: x + 0.7

3. Use the function table to graph the data on a coordinate grid.
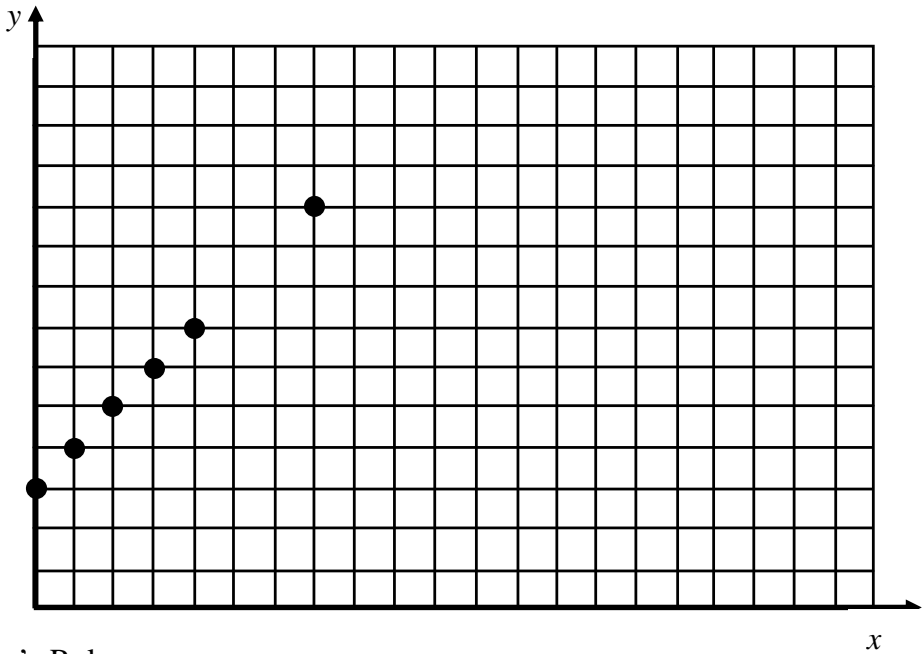
x	y
0	3
1	4
2	5
3	6
4	7
7	10

Brief Constructed Response

Step A

Complete the function table.

Rule: $2 \bullet x + 5 = y$

Input x	Output y	Ordered Pair (x, y)
2	9	(2, 9)
4	13	(4, 13)
6	17	(6, 17)
8	21	(8, 21)

Step B

- Explain why your answer is correct. Use what you know about patterns and function tables in your explanation. Use words, numbers, and or symbols in your explanation.

- *Student responses should explain how they determined their answers using what they know about function tables. The response should include knowledge of applying the rule to the x value and explaining the process of creating an ordered pair.*

   I substituted the x value in the rule with each input to get the output or y value. Then, I used the x and y values to create the ordered pair, knowing that x always comes first in an ordered pair.